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A Modified Photoacoustic Piezoelectric Model for Thermal Diffusivity Determination of Solids

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Abstract The conventional photoacoustic piezoelectric (PAPE) model did not take into account the influence of the piezoelectric transducer (PZT) on the vibrations of the sample, and this approximation brought certain error. In this article, a simple method has been proposed to investigate the vibrations of the sample–PZT combination, and the theory of the PAPE technique has been modified. By introducing an equivalent thickness parameter, the two-layered model has been simplified and an analytical expression for the phase of the PAPE output signal has been obtained. The experimental system has been set up, and the thermal diffusivities of several metal samples have been measured. The experimental results show that the modified model has a higher accuracy than the conventional model.

Keywords Photoacoustic piezoelectric technique · Thermal diffusivity

1 Introduction

In recent years, the photoacoustic piezoelectric (PAPE) technique has been employed successfully to determine the thermal diffusivities of various materials [1–9]. Based on Blonskij's simplified thermoelastic theory [1], metals [1,2], composite materials [3–6], biological tissues [7], and two-layered materials [8,9] have been studied. Nevertheless, some assumptions in the conventional theory restrict the application of the PAPE technique.

The conventional theory assumed that the piezoelectric transducer (PZT) is thin in comparison with the sample so that the influence of the PZT on the vibrations of

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the sample can be neglected and the sample can vibrate freely. Actually, since the piezoelectric ceramic material is usually hard, the interaction between the sample and PZT is considerable. When the thickness of the PZT is comparable to or larger than that of the sample, the conventional theory may bring significant error and be invalid [2,10]. Hence, the influence of the PZT becomes the main factor limiting the accuracy of the PAPE measurement.

To overcome the limitation of the conventional PAPE technique, the influence of the PZT on the sample has been taken into account. A simple method has been proposed to deal with the two-layered model (sample and PZT). By introducing an equivalent thickness parameter, the calculation of the thermoelastic problem has been simplified and an analytical expression for the phase of the PAPE signal versus modulation frequencies has been obtained.

2 Theory

The principle of the PAPE technique is as follows: a periodic heat source caused by the intensity modulated laser generates thermal waves; the thermal waves propagate in the sample and cause thermoelastic strains; and the periodic thermoelastic strains are detected by the PZT attached to the back side of the sample.

The model in the conventional theory is shown in Fig. 1. The sample and PZT are both circular plates with thicknesses of l and h, respectively. With the influence of the PZT being neglected, the model had been simplified to a single-layered thin plate. Actually, the model contains two layers and the vibration of the sample will be restrained by the PZT. In our previous work, the two-layered model had been investigated, but it brought a large mount of computation [9]. Here, we propose a simple approach to deal with the two-layered model.

The complexity of the problem lies in the difference of the elastic properties between the sample and PZT. The intensity of the restraints on the sample, under the known properties of the PZT, is determined by two factors: the thickness and the elastic properties of the sample. The thinner or more flexible the sample is, the stronger the restraints on the sample will be. Here, we assume that the sample and PZT can be treated as the same elastic material. In order to keep the same intensity of the restraints on the sample, the thickness of the PZT h should be replaced by the equivalent thickness ph, where p is a parameter to be determined. A detailed theoretical discussion on p will be carried out later.

By introducing the equivalent thickness parameter p, the difference of the elastic properties between the sample and PZT has been eliminated and the two-layered

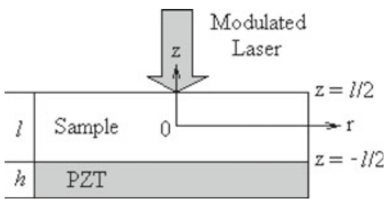

Fig. 1 Model in the conventional theory

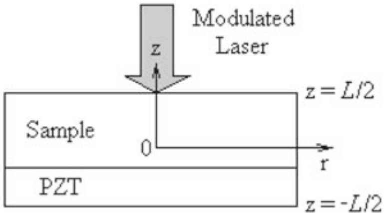

Fig. 2 Modified model under investigation

model is simplified to a single-layered model. The modified model is shown in Fig. 2, which has a thickness L, where $L = l + ph$.

The temperature distribution of the combination can be found from the heat conduction equation,

$$\Delta \widetilde{T} - \frac{1}{D}\frac{\mathrm{d}\widetilde{T}}{\mathrm{d}t} = -\frac{Q}{k}\mathrm{e}^{\mathrm{i}\omega t}, \tag{1}$$
$$\widetilde{T} = T\mathrm{e}^{\mathrm{i}\omega t},\ Q = I\beta\mathrm{e}^{-r^2/b^2}\mathrm{e}^{-\beta(L/2-z)},$$

where T is the temperature distribution, D is the thermal diffusivity of the sample, Q is the density of the heat source, k is the thermal conductivity of the sample, ω is the modulation frequency, I is the intensity of the laser, β is the optical absorption coefficient, and b is the laser radius. Here, the sample has been treated as thermally thick, and the thermal properties of the PZT make little contribution to the temperature distribution [9].

By introducing the symbol $\sigma^2 = \mathrm{i}\omega/D$ and integrating Eq. 1 over the area of the plate S, the heat conduction equation has the form,

$$\frac{\mathrm{d}^2\langle T\rangle}{\mathrm{d}z^2} - \sigma^2\langle T\rangle = -\frac{I\beta\pi b^2}{k}\mathrm{e}^{-\beta(L/2-z)}, \tag{2}$$
$$\langle T\rangle = \iint_S T(r, z)\mathrm{d}S.$$

Here, it assumes that the derivative of the temperature in the r direction should be equal to zero on the boundary of S, which is based on the fact of great damping of the thermal waves in the propagation. It should be pointed out that it is unnecessary to solve the three-dimensional heat conduction equation in the piezoelectric detection. One will see later that the piezoelectric output signal is related to the integral solution $\langle T\rangle$.

Furthermore, considering the case of strong optical absorption $\beta l \to \infty$, the integral solution $\langle T\rangle$ has the form,

$$\langle T\rangle = \frac{I\pi b^2}{k\sigma}\frac{\cosh\sigma\,(L/2+z)}{\sinh\sigma L}. \tag{3}$$

For solving the thermoelastic problem, thin plate theory has been employed [1]. Let u and w be the displacements of the plate points in the r and z directions. Thin plate theory assumes that w is independent of z, and the stress σ_z and γ_{rz} are equal to zero in all points of the plate. By using the stress-to-strain relations and the balance equations, the sum of two strains has been obtained [1]:

$$\varepsilon_r + \varepsilon_\theta = (1+v)\alpha(T_0 + z\tau) + \frac{2(1-v)\alpha}{R^2}\left(\int_0^R T_0 r\mathrm{d}r + z\int_0^R \tau r\mathrm{d}r\right), \tag{4}$$

$$T_0 = \frac{1}{L}\int_{-L/2}^{L/2} T(r,z)\mathrm{d}z, \quad \tau = \frac{12}{L^3}\int_{-L/2}^{L/2} T(r,z)z\mathrm{d}z,$$

where ν is Poisson's ratio, α is the linear thermal expansion coefficient, and R is the plate radius.

For the piezoelectric material, the governing equations are [10]

$$\begin{cases} \sigma_{ij} = C^E_{ijkl}u_{kl} - e_{kij}E_k \\ D_i = e_{ikl}u_{kl} + \varepsilon^S_{ik}E_k \end{cases} \tag{5}$$

where σ, u, E, and D are the stress, strain, electric field, and displacement; and C, e, and ε are the compliances, piezoelectric constants, and dielectric constants, respectively. Since the PZT is thin and plated on its z surface, the electric field components E_x and E_y can be neglected. Then we have

$$D_3 = e\left(\frac{\partial u}{\partial r} + \frac{u}{r}\right) - \varepsilon E_3, \tag{6}$$

where

$$e = e_{31} - e_{33}\frac{C^E_{13}}{C^E_{33}}, \varepsilon = \varepsilon_{33} + \frac{e^2_{33}}{C^E_{33}}.$$

Integrating Eq. 6 over the body of the PZT, we get

$$\iiint_V D_3\mathrm{d}V = 0 = e\int_{-L/2}^{-L/2+ph}\mathrm{d}z\iint_S(\varepsilon_r + \varepsilon_\theta)\mathrm{d}S - \varepsilon S V_\mathrm{o}, \tag{7}$$

where V_o is the output voltage and S is the surface area of the PZT. Then, the output voltage has the form,

$$V_\mathrm{o} = \frac{e}{\varepsilon S}\int_{-L/2}^{-L/2+ph}\langle\varepsilon_r + \varepsilon_\theta\rangle\,\mathrm{d}z. \tag{8}$$

Taking Eqs. 4 and 8 into account, we get

$$V_o = \frac{eph}{\varepsilon S} 2\alpha \left(\langle T_0 \rangle - \frac{l}{2} \langle \tau \rangle \right). \tag{9}$$

Here, one can observe that the output signal is related to the integral solution $\langle T \rangle$ and $\langle \tau \rangle$. The output voltage can be obtained from Eqs. 3, 4, and 9,

$$V_o = \frac{eph}{\varepsilon S} \frac{I\pi b^2}{kL\sigma^2} 2\alpha \left[1 - 3\frac{l}{L} - 3\frac{l}{L} \frac{2 - 2\cosh(\sigma L)}{(\sigma L)\sinh(\sigma L)} \right]. \tag{10}$$

By simple transforms, we obtain the expression for the phase of the PAPE signal,

$$\tan\varphi = \frac{al}{3M} \left[2 + \frac{ph}{l} - \left(\frac{ph}{l} \right)^2 \right] - N, \tag{11}$$

where $a = (\omega/2D)^{1/2}$ is called the thermal diffusion coefficient, and

$$M = \frac{[\sin a(l+ph) - \sinh a(l+ph)]}{[\cos a(l+ph) + \cosh a(l+ph)]}, \quad N = \frac{[\sin a(l+ph) + \sinh a(l+ph)]}{[\sin a(l+ph) - \sinh a(l+ph)]}.$$

The theoretical analysis shows that it is possible to determine the thermal diffusivity from the phase-to-frequency or amplitude-to-frequency characteristics. The phase-to-frequency characteristic has been demonstrated to be the better way to study the thermal diffusivity of the sample [1–9]. The thermal diffusivity D and the equivalent thickness parameter p are the only two parameters, which determine the phase-to-frequency characteristic (under the known thicknesses l and h).

Some theoretical discussion on the equivalent thickness parameter should be considered. The equivalent thickness parameter p has the same physical meaning as it does in the two-layered theory [9]. p is related to Young's modulus and Poisson's ratio of both the sample and PZT. In the two-layered theory, the phase expression contains the high-order terms of $(ph/l)^n$. This led to a complicated expression and a large mount of computation. Since *ph*/*l* is a small quantity, the high-order terms of $(ph/l)^n$ have been neglected in the modified theory, only retaining the first-order and second-order terms. In this way, the modified theory combines the conciseness of the conventional theory and the precision of the two-layered theory.

The value of p is determined by comparison of the elastic properties between the sample and PZT. When the sample is flexible compared to the PZT, p should be greater than 1, and this situation occurs when a soft material is under investigation; when the elastic properties of the sample are close to those of the PZT, p should be close to 1; when the sample is hard compared to the PZT, p should be between 0 and 1, and this situation occurs when measuring the metal sample; and when the sample is far stronger than the PZT, p is close to 0, and the modified theory agrees with the conventional one [1].

The relative error of the thermal diffusivity, calculated by the conventional and modified theories, has been studied and shown in Fig. 3. Figure 3a shows that when

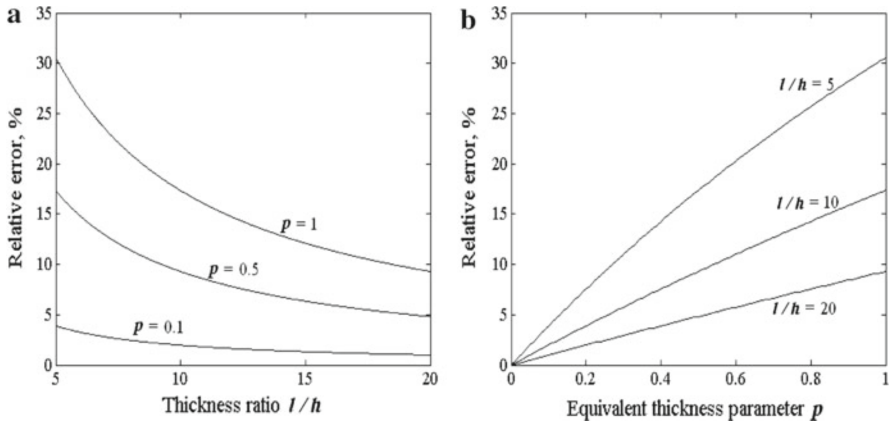

Fig. 3 Theoretical simulation on the error caused by neglecting the influence of PZT

the thickness of the PZT is closer to that of the sample, the relative error will be greater, which means that the constraints of the PZT on the sample are strong, and the PZT cannot be neglected; when the sample is much thicker than the PZT, the difference between the conventional and modified theories is negligible. Figure 3b shows that when the value of p becomes larger, the relative error will be greater, which means that when the sample is relatively flexible, the constraints of the PZT on the sample are strong, and the usefulness of the modified theory is more obvious.

3 Experiment and Discussion

The experimental setup is shown in Fig. 4. The laser produced by an argon-ion laser device (wavelength of 514 nm, power of 130 mw) is modulated by an acousto-optic modulator driven by a function generator; the modulated laser beam induced on the surface of the sample changes the temperature distribution of the sample; vibrations of the sample caused by the periodic heating are detected by the PZT attached to the back side of the sample; the lock-in amplifier deals with the output signal. The PZT is made in the form of a disk with a thickness of 0.2 mm and a diameter of 18.2 mm.

In order to verify the theoretical predictions, we have carried out experimental measurements of the phase versus modulation frequency for samples of brass, aluminum, and copper, whose thermal properties are known. The samples are made in the form of disks with diameters of 18 mm. For each set of experimental data, we use the conventional and modified theoretical curves to calculate the thermal diffusivity of the sample. The experimental data and the best-fitted curves of the modified theory are shown in Fig. 5. The comparison of measurement accuracy between the conventional and modified technique is shown in Table 1.

The p-values of the brass, aluminum, and copper samples in our experiment are calculated to be 0.54, 0.76, and 0.75, respectively. The results show that the values of p ranging from 0 to 1 are rational in terms of the metal samples.

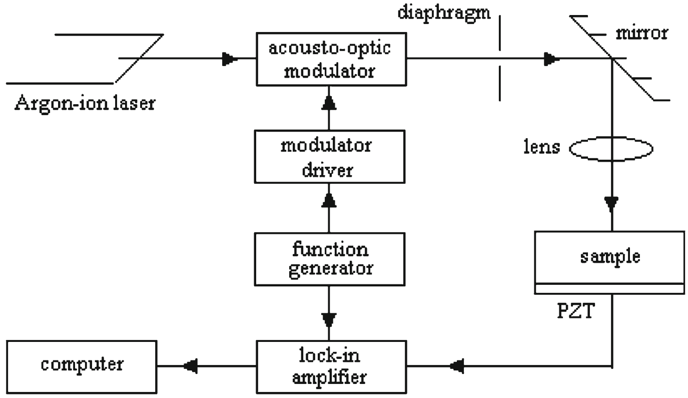

Fig. 4 Schematic diagram of experimental setup

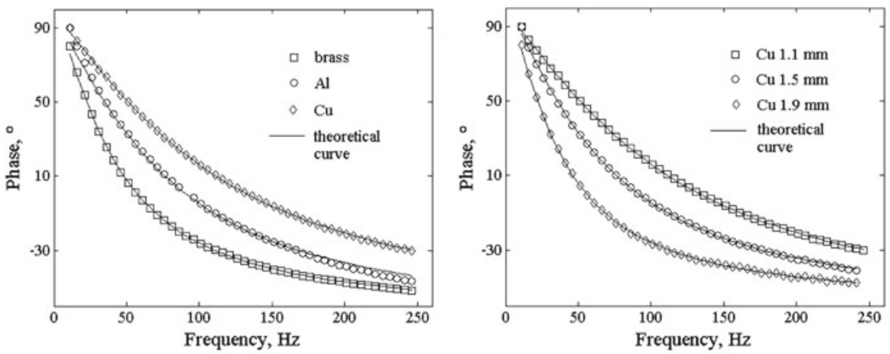

Fig. 5 Experimental data and best-fitted curves for brass, aluminum, and copper

Table 1 Comparison of accuracy between the conventional and modified technique

Sample	Thickness	Reference value [11] ($mm^2 \cdot s^{-1}$)	Conventional technique		Modified technique	
			Experimental value ($mm^2 \cdot s^{-1}$)	Error (%)	Experimental value ($mm^2 \cdot s^{-1}$)	Error (%)
Brass	1.0	38.0	34.3	9.7	38.7	1.8
Al	1.1	85.0	72.8	14.4	84.5	0.6
Cu	1.1	117.0	104.2	11.0	121.1	3.5
	1.5		114.6	2.1	118.4	1.2
	1.9		116.3	0.6	120.5	3.0

In our previous work, when the thickness of the sample is about 2 mm, the measurement error of the conventional PAPE technique is less than 5 % [2]. In Table 1, one can observe that when the thickness of the sample is about 1 mm, the conventional technique will bring significant error, while the modified one can control the error within 5 %. With the theory being modified, the PAPE technique has overcome the limitation of conventional theory and the thermal diffusivities of solid samples with various thicknesses can be investigated.

4 Conclusions

A modified theory of the PAPE technique, which takes into account the influence of the PZT on the vibration of the sample, has been developed. By introducing the equivalent thickness parameter, the two-layered model has been simplified to a single layer. The experimental results show that the accuracy of the modified PAPE technique measuring the thermal diffusivity of solids has been improved. The modified PAPE technique will have a wide range of application in determining the thermal diffusivity of various materials.

References

1. I.V. Blonskij, V.A. Tkhoryk, M.L. Shendeleva, J. Appl. Phys. **79**, 3512 (1996)
2. C.M. Gao, Y.F. Bi, Q.M. Sun, Y. Zhou, Z.P. Wang, Chin. J. Lasers **36**, 426 (2009)
3. L. Sun, S.Y. Zhang, Y.Z. Zhao, Z.Q. Li, L.P. Cheng, Rev. Sci. Instrum. **74**, 834 (2003)
4. L. Sun, S.Y. Zhang, Y.Z. Zhao, Z.Q. Li, X.J. Shui, Acta Acustica **28**, 315 (2003)
5. Y.F. Bi, Y.F. Wang, C.M. Gao, W. Li, B.X. Zhao, Piezoelectr. Acoustoopt. **30**, 601 (2008)
6. B.X. Zhao, Y.F. Wang, C.M. Gao, Y. Zhou, Z.P. Wang, Chin. J. Lasers **35**(s2), 324 (2008)
7. C.M. Gao, S.Y. Zhang, Y. Chen, X.J. Shui, Y.T. Yang, Chin. Sci. Bull. **49**, 2273 (2004)
8. M.L. Shendeleva, Proc. SPIE **3359**, 484 (1997)
9. Q.M. Sun, C.M. Gao, B.X. Zhao, H.B. Rao, Proc. SPIE **7276**, 1N (2009)
10. W. Jackson, N.M. Amer, J. Appl. Phys. **51**, 3343 (1980)
11. A.P. Smirjagin, N.A. Smirjagina, A.V. Belova, *Commercial Nonferrous Metals and Alloys Handbook* (Metalurgija, Moscow, 1974)